

Empowering Students With Computational Estimation Skills

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This paper highlights the significance of teaching estimation to students and outlines effective techniques for doing so. Estimation is a crucial skill in mathematics that supports real-world applications, improves number sense, aids problem-solving efficiency, helps identify errors, and builds student confidence. Important methods include rounding, front-end estimation, clustering, using compatible numbers, and range estimation. To embed these strategies into the curriculum, educators should incorporate regular practice, real-life problem-solving, collaborative work, the use of technology tools, and continuous assessment with reflection. Teachers, as agents of change, can significantly enhance students' mathematical abilities by emphasizing estimation, preparing them for situations requiring quick, reasonable judgments. This paper provides insights and practical guidance on integrating estimation into daily teaching, highlighting its enduring importance in education and its positive long-term impact on students' mathematical skills.

Keywords: teaching estimation, computational estimation, estimation process and strategies

Introduction

As technology advances, computers and calculators have become increasingly powerful and accessible, thereby reducing our reliance on traditional pen-and-paper calculations. Nevertheless, estimation remains a crucial skill in daily life, often serving as a practical alternative to exact computations. Whether calculating grocery expenses, estimating travel durations, or determining how much paint a room needs, people often use estimation as a quick problem-solving method. This involves approximating values when precise data are not necessary or practical, enhancing our understanding of numbers and spatial reasoning. Despite the availability of advanced computational tools, verifying the reasonableness of results through estimation is essential, particularly in math education (Tsao & Pan, 2013). As curriculum standards increasingly emphasize estimation, developing this skill is crucial to enhancing students' mathematical abilities.

As digital tools and automation transform mathematical calculations, strengthening estimation skills in education becomes more vital. While calculators offer accuracy, estimation allows quick judgments, plausibility checks, and practical application of math in daily life. Whether handling budgets, measuring materials, or analyzing data, estimation is crucial for making informed decisions. Recent research highlights the significance of computational estimation in mathematics education, suggesting that an early focus on these skills enhances problem-solving and reasoning abilities (Desli & Efstathopoulos, 2025). Moreover, adding estimation to STEM programs has been shown to enhance students' quantitative literacy and analytical skills (Suparman, Juandi, Turmudi, & Martadiputra, 2025). Therefore, educators should actively incorporate estimation into their teaching methods to increase students' confidence and skill in numerical approximation.

Meaning and Value of Estimation

Estimation is a crucial mathematical skill with practical and theoretical importance in daily life. It involves making educated guesses based on given conditions, helping students develop numerical reasoning and generate multiple plausible predictions from data. In today's society, estimation is recognized as a vital problem-solving tool that enhances decision-making and mathematical fluency in fields such as finance, engineering, and data analysis (Desli & Efstathopoulos, 2025; Suparman et al., 2025). Historically, computational estimation has received little attention in math curricula, resulting in generations of students with limited estimation skills (Bestgen, Reys, Rybolt, & Wyatt, 1980). This neglect has sparked an ongoing debate: Why is estimation considered one of the most practically useful parts of the math curriculum, yet it remains among the least effectively taught? (Bahr & Monroe, 2024). Research criticizes traditional math textbooks for mainly focusing on exact answers and often ignoring the pedagogical importance of approximation in fostering flexible thinking and problem-solving skills (R. E. Reys, Trafton, B. Reys, & Zawojewski, 1984). Many students are taught to seek a single correct answer and struggle to understand that estimation involves reasonable approximations rather than absolute precision. This problem persists because many educators do not prioritize teaching estimation, often due to doubts about its significance and a lack of structured instructional strategies (Kosova, Kap çu, Bushi, & Kosova, 2024). Ongoing research underscores the necessity for targeted interventions to enhance students' computational estimation skills (Tsao, 2013; Peeters, Degrande, Ebersbach, Verschaffel, & Luwel, 2016; Wang, 2024). By incorporating estimation activities into the curriculum and using real-world examples, educators can help students develop a deeper understanding of numerical relationships and enhance their problem-solving abilities.

Estimation plays a crucial role in mathematics education, offering benefits that extend beyond mere calculations. It involves skillfully approximating quantities or results without precise calculations, which helps students gain a deeper understanding of numbers and their application in real life. Besides practical uses in everyday tasks like shopping or budgeting, estimation also fosters critical thinking by encouraging students to analyze situations and make decisions with limited information. In educational contexts, teaching estimation adds value beyond simple arithmetic practice. It enhances students' number sense by helping them grasp the size and relationships of numbers rather than just focusing on exact figures. This mental skill not only improves mathematical reasoning but also equips learners with important problem-solving abilities across various scenarios. Moreover, estimation is key for error detection and work verification, enabling students to assess whether their answers are reasonable and identify potential mistakes. By integrating estimation into math instruction, teachers promote a balanced approach that emphasizes both accuracy and practicality.

The National Council of Teachers of Mathematics (NCTM, 1989; 2000) emphasizes the importance of estimation within its standards. Estimation is a fundamental skill that aids understanding of numbers and their relationships. According to the NCTM, estimation supports the grasping of mathematical concepts, verifying the reasonableness of answers, and making informed decisions in real life. Incorporating estimation into the math curriculum involves:

- (1) Developing Number Sense: Students learn to make reasonable guesses about quantities and measurements, which boosts their ability to work with numbers flexibly.
- (2) Encouraging Mental Math: Estimation fosters mental calculation strategies, enabling students to solve problems more efficiently without depending solely on calculators.

(3) Applying Practical Contexts: Students participate in activities that involve estimation in real-life situations, such as shopping, cooking, and planning.

(4) Enhancing Problem-Solving Skills: Through estimation practice, students sharpen their ability to analyze problems, make predictions, and evaluate the plausibility of their answers.

The NCTM standards highlight that estimation is more than just a preliminary step before exact calculations; it is a valuable skill. They encourage teachers to incorporate estimation activities into their lessons to help students develop a strong math foundation and enhance their problem-solving confidence. The significance of estimation lies in its use across various fields and its role in developing students' analytical abilities and understanding of uncertainties in quantitative reasoning. By emphasizing estimation, teachers equip students to confidently and effectively handle the complexities of a data-driven world.

Estimation Process and Strategies of Computational Estimation

Numerous estimation strategies provide simplified calculation methods, making mental math easier. These approaches enable quick, approximate values to be obtained from a calculator. Users can choose different methods based on their preferences or thinking style and often combine multiple strategies when solving problems. The following overview highlights the key processes and estimation techniques outlined by researchers.

Estimation process. Reys, Bestgen, Rybolt, and Wyatt (1982) conducted an influential study on the estimation process, examining how people adopt different strategies for approximate calculations. Their research provided valuable insights into the cognitive mechanisms behind estimation and underscored the importance of flexibility in applying various methods. The study revealed common strategies and behaviors employed by individuals when approaching estimation tasks, highlighting that combining multiple strategies can enhance both accuracy and efficiency. This research has had a significant impact on mathematics education, influencing teaching methods and curriculum design to enhance students' estimation skills.

Reformulation. We can simplify numbers into easier structures to facilitate mental calculation, while preserving the essence of the original equation. For example, estimating " $789 + 104$ " can be made easier by rounding to " $800 + 100$ ". Rounding is one method, but using "nicer numbers" works as well. For instance, you could approximate " $73 \div 9$ " as " $72 \div 9$ ", which equals 8, or estimate 34% as roughly 3. Restructuring often involves using approximate values to turn complex problems into more straightforward mental calculations.

Translation. Adjust the structure of the estimation problem to a form that's easier to compute mentally or familiar. There are two types of translation: first, changing the equation itself, such as estimating $3052 + 2988 = ?$, which can be simplified to " $3000 + 3000$ ", then to $3000 \times 2 = 6000$; second, keeping the structure the same, like estimating $(347 \times 6) \div 43 = ?$, which can be adjusted to $(350 \times 6) \div 42 = 350 \times (6 \div 42) = ?$, simplifying to $350 \times (6 \div 42) = 350 \times 1/7 = 50$.

Compensation. During restructuring or translation processes, it is advisable to moderately adjust the size of numerical values to account for estimation errors, making the estimated answers more plausible and closer to the actual values. Phrases like "more", "less", "not more", and "not less" can also be added to enhance the estimates. Compensation can be classified into two types: (a) during calculation compensation—for example, when solving $891 + 960 + 1040 + 996$, rounding 960, 1040, and 996 to 1000, and 891 to 900 results in a sum of 3900 instead of 4000, providing a more accurate approximation of the actual total than treating all numbers as 1000; and (b) post-calculation compensation—for instance, when evaluating $1000 \div 24$, it can be approximated

as $1000 \div 25 = 40$. Since 24 is slightly less than 25, the actual quotient exceeds 40; therefore, adjusting the answer to 41 will provide a more precise estimate.

Strategies for Computational Estimation

Regarding estimation techniques, rough estimation is a common approach. The author first presents the “rough estimate” strategy and then, referencing research by Dowker (1992), Reys et al. (1982), Schoen, Friesen, Jarrette, and Urbatsch (1981), and Tsao (2009), summarizes 10 additional estimation methods.

Rough Estimation. “Rough estimation” involves calculating an approximate number that is reasonably close to the actual value. The process includes modifying a number through rounding, truncation, or ceiling techniques to meet specific needs. While the final estimate may differ from the exact number, it effectively fulfills its purpose. Here are the standard approximation methods:

(1) **Rounding Method:** For example, estimating 24,687 and 36,892 can be done by rounding 24,687 to 25,000 and 36,892 to 37,000 using “rounding to the nearest thousand”. Summing these gives 62,000, illustrating how rounding simplifies complex calculations.

(2) **Truncation Method:** For instance, estimating the sum of 45,789 and 67,234 can be achieved by truncating 45,789 to 45,000 and 67,234 to 67,000 using “truncation to the nearest thousand”. Their sum is 112,000, illustrating how truncation facilitates addition by focusing on the leading digits.

(3) **Ceiling Method:** For example, estimating $13,374,236 + 15,826,521$ by adjusting these to 14,000,000 and 16,000,000, respectively, using “ceiling to the nearest hundred thousand”. The total is 30,000,000, demonstrating the use of the ceiling method for estimation.

Front-end strategy. This method simplifies numbers by highlighting their leading digits and adjusting them before mental calculation. For instance, to compute “ $3,209 + 4,887$ ”, round each number to 3,000 and 5,000 using the “rounding of leading digits” method, then sum to get 8,000.

Improved front-end strategy. This method is more flexible than the front-end approach because it separates the leading digits from the rest of the number, processes them individually, and then combines the results for a more accurate estimate. For example, estimating $869 + 432$ involves first using the front-end strategy to get $800 + 400 = 1,200$, then handling the remaining parts to approximate $70 + 30 = 100$, and finally combining these to estimate around 1,300.

Clustering strategy. This process involves spotting numbers close to particular values (such as averages) in an equation and using them for approximation. For instance, estimating “ $3,121 + 2,988 + 2,995$ ” reveals all numbers are near 3,000, so the total is approximately $3,000 + 3,000 + 3,000 = 9,000$ or $3,000 \times 3$.

Rounding strategy. This method involves rounding numbers to simplify calculations, often by slightly overestimating one number and underestimating another. For example, rounding 247 to 250 and 519 to 500 results in a total of 750. This technique aligns with the “rounding two numbers” and “rounding one number” strategies.

Compatible number strategy. Select numbers that make calculations easier. For instance, estimating $32 + 46 + 60 + 53$ by pairing $32 + 60 \approx 100$ and $46 + 53 \approx 100$, giving a total of about 200. Alternatively, for $3,628 \div 12.1$, round to $3,600 \div 12$ to estimate 300, or estimate 49×5 by rounding 49 up to 50 for a calculation of $50 \times 5 = 250$.

Special numbers strategy. People employ different methods to manage various number formats, converting them as necessary. For instance, 26% or 0.258 is approximately equal to $1/4$.

Conversion between numerals. This includes conversions such as: (a) $66 \div 0.86 \approx 66 \div 6/7 \approx 66 \times 7/6 = 77$; (b) 34% is roughly 0.30, close to $1/3$; (c) 11% is about 0.10, approximately $1/10$; (d) 51% is near 0.5, which equals half.

Using known or nicer numbers. Use familiar results to estimate. For instance, approximately $1,867 \div 30$ by using $1,800 \div 30 = 60$. Alternatively, estimate 198×52 by using $200 \times 50 = 10,000$, which simplifies the calculation.

Rounding two or one number. In multiplication, round both numbers to the nearest power of ten; for instance, 145×38 becomes 150×40 , resulting in 6,000. Alternatively, round one number while keeping the other the same, such as approximating 63×78 by rounding 78 to 80, giving 63×80 . These estimation techniques improve speed and accuracy by simplifying complex calculations. Using methods such as rounding, front-end estimation, clustering, and benchmarking enables quick and informed approximations through various strategies. These approaches help students develop number sense and mental calculation skills in educational contexts.

Teaching Estimation

Estimation has often been overlooked in math education. Research indicates that teaching estimation can improve students' skills in this area (Tsao & Pan, 2013). Incorporating estimation tasks can also deepen understanding of how mathematical concepts, calculations, and problem-solving interconnect. Teaching estimation encourages students to grasp number concepts and processes, developing skills in estimation techniques, place value, numerical magnitude, factual number concepts, and various arithmetic meanings (Rubenstein, 1985; Tsao, 2009; Tsao & Pan, 2013). Unlike other math topics, estimation is not usually given dedicated units in textbooks. Teachers can integrate estimation with other concepts to enrich learning. First, teachers need to recognize the importance of estimation. Then, they should purposefully connect estimation with real-world problems, guiding students step by step. Consistent practice is essential for improvement in estimation, just as it is for other skills. Focusing only on isolated units may limit estimation to small parts of calculations, leading students to adopt rigid strategies (Sowder, 1984). Teachers play a key role by motivating students, designing suitable curricula, and applying specific teaching strategies: (1) Promoting estimation language such as "approximate", "more than", "less than", "around", "above", "below", "exceeding", "not exceeding", "less than", "not less than", "greater than", "not greater than", "at most", "at least", etc. (2) Introducing estimation through everyday contexts to highlight its relevance. (3) Using students' real-life experiences broadly. (4) Starting with diverse examples and accepting different estimation methods. (5) Using group discussions to teach estimation and validate various student answers. (6) Helping students grasp effective strategies to develop their estimation abilities. (7) Assisting students in choosing appropriate strategies, understanding needed accuracy, and judging the reasonableness of estimates (Trafton, 1986; Hope, 1989; Tsao, 2009; Tsao & Pan, 2013). Teachers should also guide students on applying estimation to solve math problems. First, teach prediction techniques so students can reasonably estimate result ranges and recognize errors if their answers fall outside these estimates. Encourage a thorough understanding of problem data and intuitive thinking to guide correct calculations. Second, teach adjustment strategies, since different estimation methods can vary in accuracy. Proper adjustments help estimates align more closely with real values. Since estimation values do not need to be exact, there are no right or wrong answers—only reasonable methods and applications. Use practical examples to demonstrate the usefulness of estimation, avoiding excessive comparisons with precise calculations to foster versatile thinking. Teachers must understand various estimation strategies and students' capabilities, gradually

transitioning from real-life examples to more abstract methods, so that students can apply these skills daily. Recognize that students may employ different estimation approaches, resulting in varied outcomes. Respect their perspectives, encourage independent thinking, facilitate discussions on different strategies, and promote peer and self-evaluation. Support any reasonable estimation method that captures the core of estimation rather than focusing solely on accuracy.

Integrating estimation into teaching is a valuable practice in mathematics education for several reasons:

(1) **Enhancing Estimation Skills:** Evidence indicates that explicit instruction in estimation can significantly boost students' accuracy in guessing. Regular practice of estimation tasks helps students develop better numerical intuition and sound judgment when approximating values.

(2) **Deepening Conceptual Understanding:** Teaching estimation aids students in understanding the relationships between mathematical concepts. For instance, estimating the outcome of an arithmetic operation requires considering the sizes of numbers and their relative scales, reinforcing number sense and mathematical connections.

(3) **Linking Procedural and Conceptual Knowledge:** Estimation bridges procedural calculations and conceptual comprehension. While precise calculations are crucial, the ability to estimate enables students to check the reasonableness of their answers and grasp the core ideas behind the procedures.

(4) **Supporting Problem-Solving Skills:** Estimation is vital for practical problem-solving, allowing students to quickly evaluate whether answers seem plausible, which is helpful in real-world contexts where exact answers may not be necessary. It also fosters flexible thinking and adaptability.

(5) **Developing Number Sense:** Estimation helps students appreciate numerical magnitude, place value, and the relative size of numbers. They learn to make informed judgments and understand how numbers interact in various contexts.

(6) **Promoting Practical Use:** Estimation is a practical skill applicable to everyday tasks such as budgeting, shopping, measuring, and cooking. Including estimation in the curriculum highlights its relevance to daily life.

(7) **Using Different Estimation Techniques:** Teaching diverse methods, such as front-end estimation, rounding, and benchmarks, provides students with multiple strategies for various scenarios. This variety enhances adaptability and confidence in math skills.

(8) **Building Confidence and Lowering Anxiety:** When students realize they can make reasonable guesses without needing exact answers, it can reduce anxiety and boost confidence, especially for those intimidated by precise calculations.

Overall Educational Impact

Incorporating estimation into teaching enhances the educational experience by fostering a comprehensive understanding of mathematics. It equips students for academic success and practical problem-solving. Overall, incorporating estimation into math instruction offers numerous benefits, including enhancing estimation skills, deepening comprehension of mathematical concepts, and improving problem-solving abilities. By emphasizing estimation, teachers can help students develop a well-rounded mathematical foundation that is valuable both in school and in real-world situations.

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